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Two aspects of optimal diet determination for pig production: efficiency of solution and incorporation of cost variation

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Abstract For many decades linear programming has been used to find minimum cost diets, notably in the chicken and pig meat industries. More recently, animal growth models together with nonlinear optimisation methods have been used to find feeding schedules which simultaneously minimise feed costs and maximise market return, so maximising gross margin. Genetic algorithms can handle these problems, albeit slowly. In this paper we study the particular nature of the objective function (for pig meat production) and develop a global optimisation algorithm tailored to its discontinuous structure. We also demonstrate the use of stochastic programming to cope with changing feed costs and changing price at slaughter.

Keywords Feeding schedule · Gross margin · Growth model · Ingredient schedule · Linear programming · Nonlinear optimisation · Price schedule · Stochastic programming

AMS Subject Classification (2000) 90C26 · 90C56 · 90C59 · 90C90

1 Introduction

Efficient pig meat production is of critical importance on our increasingly finite planet. For many decades, linear programming has been used to determine minimum cost pig diets, based on a range of feedstuffs, their cost, their composition and dietary constraints. With the advent of pig growth models [3] and nonlinear optimisation algorithms, it is now possible to extend this traditional use of optimisation to determine a feeding schedule which maximises profitability [2]. This problem has also been addressed in [8] and [10].

The purposes of this paper are twofold. First, we explore the nature of the objective function and describe an algorithm which is tailored to its form and which moves to the optimum more rapidly than the genetic algorithm used in [2]. Second, feed costs and price received

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at slaughter are subject to variation in time. We show how to include such variation in our modelling and handle it using stochastic programming.

The paper is organised as follows. In the next section we background the objective function to be maximised. The particular shape of the objective function is explored in Sect. 3 and a Tailored algorithm introduced which finds the objective function maximum. Numerical results are presented. Variation in costs and prices are modelled in Sect. 4 and runs summarised. Section 5 contains a summary and discussion.

2 Background

In this section the domain of the objective function and the objective function itself are described. This extends an earlier presentation given in [2].

2.1 Domain

An animal "diet", specifying nutrition for a certain growth period, and required as input to a simple growth model, can be described using only three parameters, p, r and d, defined as follows:

p = proportion of the *ad libitum* digestible energy intake
r = lysine to digestible energy ratio, in grams per MegaJoule
d = digestible energy density, in MegaJoules per kilogram

Typical parameter ranges used are [0.8, 1] for p, [0.2, 1.2] for r and [12, 17] for d. The *ad libitum* digestible energy intake is determined by a standard National Research Council curve [9], relating digestible energy to live weight (LW) of the animal. Parameter p determines the proportion of that amount to be fed. Lysine is an essential amino acid, required for growth, and generally the first amino acid found to be limiting in a diet. For that reason we specify the level of lysine required using parameter r and specify the other amino acids needed for growth (in a particular ratio to lysine, providing so-called "ideal balanced" protein) using the lysine level. Finally, the interval constraining energy density d of the diet reflects the range of existing values in the ingredients.

In general, by a "feeding schedule" we refer to a finite sequence of diets, $(p_k, r_k, d_k), k = 1, \ldots, K$, with the kth diet fed for T_k days, $T_1, \ldots, T_{K-1}$ fixed at the outset and T_K chosen less than or equal to $T - \sum_{k=1}^{K-1} T_k$ (where T is an upper bound for the total feeding period) to maximise profitability for the given diets. New Zealand examples used in this paper will employ two diets, namely "grower" and "finisher" diets, with $T_1 = 35$ and T typically 105 days. We write such a feeding schedule, with the time periods understood, as

$$F = (p_1, r_1, d_1; p_2, r_2, d_2)$$

The total period of 105 days, for New Zealand conditions, amply covers the usual time from weaner arrival to slaughter. Our aim will be to find the optimal feeding schedule and slaughter date, so the domain of the problem is

$$P_1 \times R_1 \times D_1 \times P_2 \times R_2 \times D_2 \times \{T_1 + 1, \ldots, T\}$$

where $P_k = [0.8, 1]$, $R_k = [0.2, 1.2]$ and $D_k = [12, 17]$ for $k = 1, 2$.

We pause for some practical comments. On a commercial pig farm today, around the world, pigs are fed two or sometimes three diets (growers, and finishers) during the time

Table 1 Ingredient schedules IS_1 and IS_2 used in the paper: these comprise a list of ingredients and the associated costs in $/kg

Ingredient	Barley	Blood meal	Soybean meal	Maize	...	Tryptophan
Costs for IS_1 ($/kg)	0.2	1.5	1.2	2.0	...	20
Costs for IS_2 ($/kg)	1.0	0.4	0.2	0.5	...	20

The first ingredient schedule is carbohydrate cheap and protein expensive while the second ingredient schedule is carbohydrate expensive and protein cheap

from 20 kg to slaughter; this is the situation investigated in this paper, using ingredients, their cost and the price schedule for New Zealand conditions. With advanced technology, however, it will be possible to change diets more often (weekly or even daily, with phase feeding). In [2] we investigated the effect of changing diets more frequently, on a weekly basis, as the methods existed to do so.

2.2 Objective function

An objective function to be maximised is profit, or gross margin per pig, given by

$$g(F) = \max_x g(F, x) \quad \text{where}$$
$$g(F, x) = \text{Gross Return}(F, x) - \text{Feed cost}(F, x) - \text{Weaner Cost}$$

with F a feeding schedule and x the number of days until slaughter. Alternatively, and throughout this paper, we prefer to measure the more practical "gross margin per pig place per year" GMPPY. This is computed as $\max_x f(F, x) = \max_x (365/(x + 7))g(F, x)$ (when there is a seven day turnaround between batches).

The weaner cost *WC* is fixed, typically in New Zealand at NZ$70. Feed cost *FC* is the minimum feed cost given F (determined using linear programming), for the period of x days; this requires use of a schedule of ingredient costs, as shown in the examples in Table 1. Gross return *GR* is determined by the backfat thickness and carcass weight of the pig, which in turn are determined by F and x. This requires a schedule of prices, as shown in the example in Table 2. Succinctly, $g(F, x)$ is given by

$$g(F, x) = GR(F, x) - FC(F, x) - WC$$

An iteration of the routine uses r_k and d_k to complete the right-hand-side constraints in the linear programme; the least cost makeup of 1 kg of feed for this period is the output. Together with p_k and the standard NRC feed intake curve [9] this allows the feed cost for this kth period to be computed. The amount of balanced amino acid can also be calculated. This, together with the genotype parameters (and at the start the initial mass P_0 of protein in the pig) and the growth model, allows us to grow the pig for the kth period. Protein and lipid deposition are recorded. Overall growth allows us to compute $FC(F, x)$ (by summing the individual period feed costs) and $GR(F, x)$ (by referring the configuration of the pig at slaughter date to the price schedule). Pig genotype parameters in the growth model which thence influence the objective function are $Pd_{\max}$, the maximum daily protein deposition, and min LP, the minimum allowable lipid to protein ratio.

Thus the objective function of interest is calculated in two steps:

1. Calculation of $f(F, x)$, the gross margin per pig place per year when feeding schedule F is administered for x days.

Table 2 Price schedule 1 (PS_1): a New Zealand price schedule giving prices in cents/kg for pigs at slaughter in July 2001

Backfat (mm)	Carcass weight (kg) 35.0 and under	35.1 – 40.0	40.1 – 45.0	45.1 – 50.0	50.1 – 55.0	55.1 – 60.0	60.1 – 65.0	65.1 – 70.0	70.1 – 75.0	75.1 – 80.0	80.1 and over
<6	300	300	300	300	300	300	300	300	300	300	300
6–9	360	385	395	395	385	370	370	370	370	365	335
10–12	360	385	385	390	375	370	370	370	370	365	335
13–15	330	330	330	330	330	335	335	335	335	330	305
16–18	260	260	260	260	260	270	270	270	270	270	270
>18	230	230	230	230	230	240	240	240	240	240	240

2. Determination of the maximum gross margin per pig place per year for feeding schedule F, namely $f(F) = \max_x f(F, x)$.

Our aim now is to determine the maximum gross margin per pig place per year over all feeding schedules, using non-linear optimisation, as $\max_F f(F)$.

3 Tailoring a maximisation algorithm to the objective function

Even for modestly sized problems, pure random search is not able to find the optimum in a reasonable length of time. A number of standard optimisation techniques have been applied to this problem, namely Tabu search [5], simulated annealing [1], the Nelder-Mead simplex algorithm [11], adaptive random search [6] and a genetic algorithm [7,12]. The genetic algorithm has been found to be most successful to date. The "genome" is the feeding schedule, while crossover and mutation act in a natural way; associated software, named "Bacon Max" has been developed in Visual C++. Extensive testing to tune the genetic algorithm has been conducted. Generally a population size of 20 feeding schedules has been found to be satisfactory and 20 iterations with no change in the value of the objective function has provided a suitable stopping rule. The genetic algorithm is slow, however, and a very large number of feeding schedules must be searched. It takes up to an hour on a standard PC to satisfactorily solve a typical problem.

In practice, we would like to be able to solve this problem, with all the variations created by parameter changes, far more rapidly. For this reason we have explored the very particular form of the objective function and tailored a method to the finding of the maximum. This method climbs the objective function quickly at the beginning, compared with a genetic algorithm. The method is unashamedly a heuristic, deserving attention thanks to the practical importance of this problem type.

The idea behind the algorithm arose from a classification of cross sections through the optimal solution; typical such cross-sections are shown in Fig. 1. These reveal a single, craggy peak. The cross-sections exhibit different slopes and the peaks are not always central. Pig genotype parameters, $Pd_{\max}$ and min LP influence the level (and shape) of the objective function. The discontinuities in the function are attributable to discrete changes in x, the number of days for which the pig is grown, together with passage of the grown pig (based

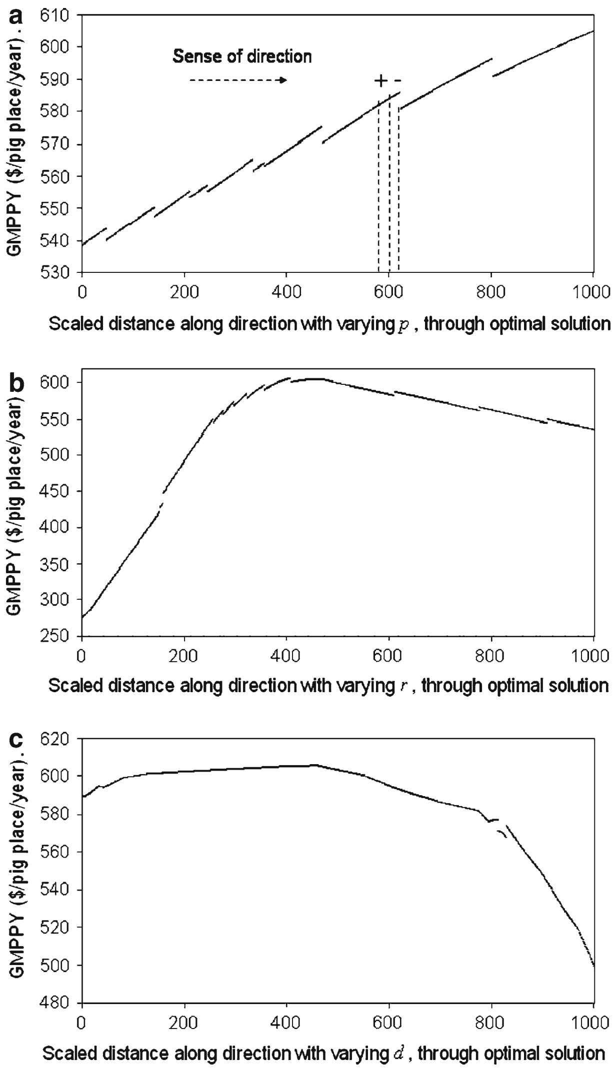

Fig. 1 Cross-section types that motivated the look-up table

on backfat thickness and carcass weight change) from one cell of the discontinuous price schedule to another.

We set out to develop a sequential hill-climbing algorithm tailored to the characteristics of this very particular objective function. It can be seen from the sections shown in Fig. 1 that it is sometimes wise to move downward in the short run, since this can lead to the overall peak; picking the direction in which to move is the challenge. Study of the cross-sections suggests use of comparison of "very close" and "close" function values in order to determine in which direction to step. Experimentation revealed that use of 0.05 and 0.10 of the distance between the current feeding schedule and the edge of the enlarged domain was successful. (The domain is enlarged in order to incorporate reflection into the search, so avoiding jamming in domain corners, as advocated in [13].) The decision regarding sense (forward

Table 3 This table is used to determine in which sense along the new direction we step. Choose rows, using the signs of ΔF_1 and ΔF_2 and columns two and three

Case	Sign		Size		Decision
	ΔF_1	ΔF_2	ΔF_1	ΔF_2	
1	+	+	S/L	S/L	Positive
2	+	−	S	S	Positive
3	+	−	S	L	Positive
4	+	−	L	S	Positive
5	+	−	L	L	Negative
6	−	+	S	S	Positive
7	−	+	S	L	Negative
8	−	+	L	S	Positive
9	−	+	L	L	Positive
10	−	−	S/L	S/L	Negative

Choose further within rows if necessary using the magnitude of the differences (with experience showing that the dividing point between small S and large L is \$3/pig place/year for New Zealand data). The rightmost column indicates positive sense or negative sense

or backward) along this direction is then made based on the sign and size of the objective function difference between these two points. Details are shown in Table 3. We generate three such random search directions at every iteration, and choose that indicating the largest positive gradient.

After the decision has been made to move in a particular sense (positive or negative) of a given direction, a move is made to the middle of the two calculated points, so 0.075 of the distance from the feeding schedule to the edge of the enlarged domain. In every iteration this step size is reduced by a “shrink factor”, S. This has been tuned and found to work successfully when $S = 0.9 \times e^{-0.001 \times iter}$, where $iter$ is the iteration counter. After many iterations the step size will reduce substantially and allow the algorithm to move close to the edge of the domain.

The three parameters comprising each diet (p, r and d) are on very different scales, since $P_k = [0.8, 1]$, $R_k = [0.2, 1.2]$ and $D_k = [12, 17]$. Standardisation to a unit interval of each parameter ensures that the search spreads over the domain. The enlarged standardised domain is then a product of intervals $[-0.5, 1.5]$ of twice the width.

The stages of the Tailored method are now described. We use two diets, so six real dimensions.

3.1 Tailored method

1. Generate initial feeding schedule. Generate the initial current feeding schedule, on the enlarged domain, using values for each coordinate drawn independently from a uniform distribution on $[-0.5, 1.5]$, giving

$$F'' = (p_1'', r_1'', d_1''; p_2'', r_2'', d_2'')$$

   Set $iter = 0$ and $S = 1$.
2. Generate candidates for the next feeding schedule.

2.1 Generate directions for progress. Set $iter = iter + 1$. Randomly draw three directions, D_1, D_2 and D_3, with each component of each direction drawn from a standard normal distribution, as in [14]. Normalise these directions.

2.2 Calculate two nearby feeding schedules for each direction. Calculate the feeding schedules at 0.05S and 0.1S of the distance from the current feeding schedule to the edge of the enlarged domain in the positive sense of each direction (six points).

2.3 Reflection into the standardised domain. For each feeding schedule outside the standard domain (including possibly the current one), reflect into the standard domain using

$$y = \begin{cases} -x, & \text{if} \quad x < 0 \\ x, & \text{if} \quad x \geq 0 \text{ and } x \leq 1 \\ 2 - x, & \text{if} \quad x > 1 \end{cases}$$

where x is a component of F'' and y now constitutes the corresponding component of a standardised feeding schedule F'.

2.4 Back transform all standardised points. Back transform the six feeding schedules and the current feeding schedule from the standardised form after reflection to the original domain, where $P_k = [0.8, 1]$, $R_k = [0.2, 1.2]$ and $D_k = [12, 17]$, via $p_k = 0.2p'_k + 0.8$, $r_k = r'_k + 0.2$ and $d_k = 5d'_k + 12$. We then have six feeding schedules and the current feeding schedule in the form $F = (p_1, r_1, d_1; p_2, r_2, d_2)$.

3. Calculate objective function values at these feeding schedules.

3.1 Calculate minimum feed cost. Use linear programming to find minimal cost diets for each diet in the six new feeding schedules.

3.2 Grow the pig for x days ($x = 1, \ldots, T$) using the pig growth model [4] and calculate the backfat thickness (mm) and carcass weight (kg).

3.3 Gross return and gross margin. Find the price of pig at slaughter in the price schedule and calculate the gross margin per pig place per year (GMPPY) for each feeding schedule and each x. Maximise over x and record the objective function value $f(F)$ for each feeding schedule.

4. Choose best direction and sense.

4.1 Calculate objective function changes in the positive sense of each direction. Calculate
ΔF_1 = GMPPY at 0.05 point − GMPPY at current
ΔF_2 = GMPPY at 0.1 point − GMPPY at 0.05 point
ΔF_3 = |GMPPY at 0.1 point − GMPPY at current|

4.2 Choose next direction and sense. Choose the next direction as the steepest, that producing the maximum value of ΔF_3. Move in the forward or backward sense in this direction, based on the decision criteria in Table 3. Figure 1 showed varying patterns for the objective function cross sections through the optimal feeding schedule. Table 3 responds to this pattern, by providing rules for progress. For example, Fig. 1a displays a small positive value for ΔF_1 and a larger negative value for ΔF_2. This is Case 3 in Table 3, so we decide to move in the positive sense of this direction.

5. Move to next point. Move in the positive or negative sense on the steepest direction by $0.075S$ of the distance from the current feeding schedule to the edge of the enlarged domain. This new point becomes the next current feeding schedule, F''.

6. Stopping rule. Set $S = 0.9 \times e^{-0.001 \times iter}$. We slow movement by this shrink factor S as the algorithm progresses. This allows the current point to progressively move toward the domain boundary, if necessary. Return to Step 2. (If the objective function does not

Table 4 Numerical results for a comparison of the performance of the Tailored method and the genetic algorithm

Results	Live weight constraint (kg)			
	84≤LW≤86		LW>80	
	GA	Tailored	GA	Tailored
Average GMPPY($/pig place/year)	578.67	578.58	579.96	579.60
Average number of feeding schedules evaluated	18560	3872	17418	1994
Average running time	90 mins	72 mins	63 mins	38 mins
Standard deviation of running time	55 mins	15 mins	41 mins	10 mins

Ten runs were used in each of the four cases. Each run was stopped when an objective function of $578/pig place/year was reached and unchanged for 20 iterations

improve in 10 iterations, return to the previous current feeding schedule and return to Step 2.) Stop after a preset number of iterations (usually 3,000).

3.2 Numerical results

The numerical results, shown in Table 4, allow us to compare the efficiency of the genetic algorithm with that of the Tailored method on the basis of average GMPPY, average number of feeding schedules evaluated and average and standard deviation of running time using 10 runs. This study was performed on an Inspiron 6000 laptop possessing an Intel® Pentium® M processor (1.73 GHz) with 504 MB of RAM. The operating system was Microsoft Windows XP Professional. Visual C++ was used in the study of both methods. The genetic algorithm in Bacon Max, however, was processed using a Windows application and the Tailored method was processed using a Console application. Pig genotype parameters, $Pd_{\max}$ and min LP were set to 160 g/day and 0.8 respectively for a single pig in the objective function.

Two situations of live weight at slaughter have been included in this study. Firstly, a live weight at slaughter of 84–86 kg has been chosen which is consistent with the slaughter weight used in New Zealand. Secondly, a live weight of more than 80 kg was chosen to examine the efficiency of both methods on a wider range of weight sampling. The price schedule from July 2001 in Table 2 has been used for these calculations. The best solution from a genetic algorithm following 20 iterations (predetermined for this study) was a GMPPY of $578/pig place/year and was chosen as the stopping criteria for the Tailored method in this section.

Figure 2 and the results from Table 4 provide a typical comparison of the Tailored method with that of the genetic algorithm. An analysis of the results indicates that the Tailored method performs better, using a significantly smaller number of feeding schedules than the genetic algorithm, for both live weight constraints. Further, with regard to the average running time, the Tailored method was found to be slightly faster than the genetic algorithm for live weight constraint 84–86 kg, but significantly faster when the live weight constraint was more than 80 kg. A possible reason for this emerges from an examination of the cross-sections through the optimal value of the objective function under the differing constraints, shown in Fig. 3. For a live weight constraint of 84–86 kg, the corresponding carcass weight is around 65 kg. This yields a relatively narrow objective function peak, as illustrated in Fig. 3a. On the other hand, for a live weight constraint of more than 80 kg, carcass weight is greater than 60 kg.

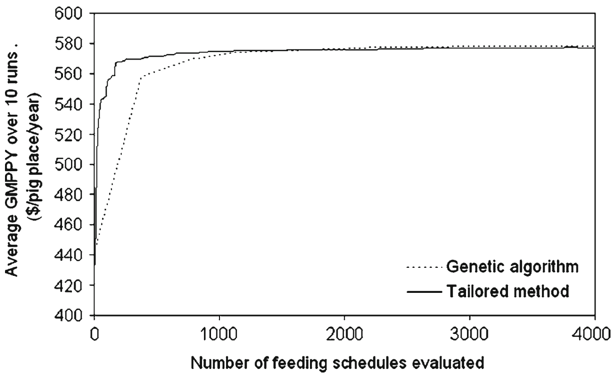

Fig. 2 A comparison of the performance of the Tailored method and the genetic algorithm for live weight constraint 84–86 kg

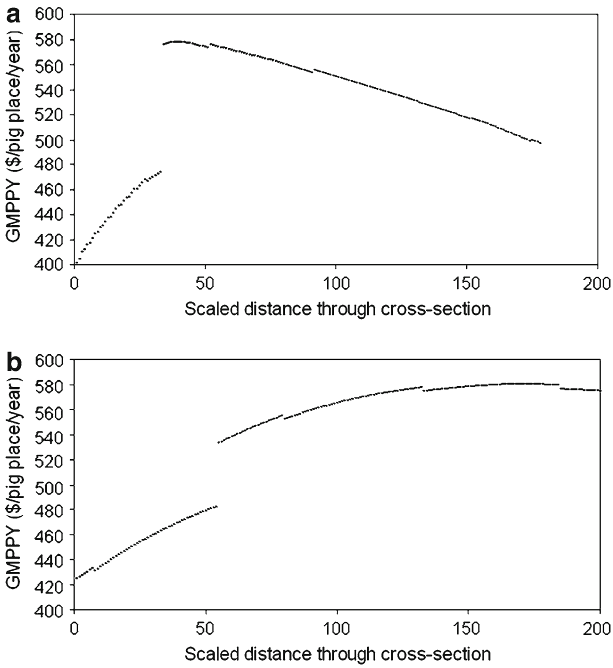

Fig. 3 Cross-sections through the optimal value of the objective function when (**a**) live weight is constrained to be 84–86 kg and (**b**) live weight is constrained to be more than 80 kg

This produces a relatively broader peak of the objective function, as seen in Fig. 3b. For this reason, the Tailored method performs more effectively on the second problem.

We conclude that the Tailored method is more efficient than the genetic algorithm, using fewer feeding schedule evaluations and shorter running times. We have shown that the improvement in performance can depend on the pig genotype parameters and the constraints.

4 Varying feed cost and price schedule

So far we have regarded the feed costs and price schedule as fixed. In practice, a producer faces uncertainty in both the future cost of ingredients (in the form of the ingredient schedule *IS*) and the price received for a pig (in the form of the price schedule *PS*). We assume now that we have I feed ingredient schedules IS_i and J price schedules PS_j occurring with probabilities p_i (in any period) and p'_j, for $i = 1, \ldots, I$ and $j = 1, \ldots, J$ respectively. Note that $FC(F, x)$ is influenced by IS_i and $GR(F, x)$ by PS_j. Note also that we do not change the feed ingredients, but vary only the cost of the feed ingredients. In order to simplify the immediately following presentation, but with no real loss in generality, we assume again that each feeding schedule comprises two diets, so $F = (D_1, D_2)$.

When finding the first diet, the ingredient schedule is fixed because we know the ingredient cost at that time, but for the second diet, the ingredient schedule will be subject to variation. Thus the producer is faced with a two-stage decision making process. Bellman's principle of optimality makes clear that an optimal strategy must be optimal at each stage (using the outputs from the previous stage); we now separately consider these two stages.

At the outset, a decision must be made about D_1. This is made facing uncertainty in IS_2 and GR (via the price schedule), so is chosen to be D'_1, the argument maximising the expected gross margin per pig place per year, through calculation of

$$\max_{D_1, D_2} \max_{x} \; c(x) \left\{ \sum_{j=1}^{J} p'_j GR_j(D_1, D_2; x) - FC_1(D_1; x) - \sum_{i=1}^{I} p_i FC_{2i}(D_2; x) - WC \right\}$$

where $c(x) = 365/(x + 7)$. Here $GR_j(D_1, D_2; x)$ is the gross return using price schedule j and diets D_1 and D_2 for a total of x days, $FC_1(D_1; x)$ is the minimum feed cost using diet D_1 in feed period one for x days and $FC_{2i}(D_2; x)$ is the minimum feed cost using diet D_2 and ingredient schedule i in feed period two, when growth is for a total period of x days.

At the second stage, given diet D'_1 in period one and the now known second period ingredient schedule IS_i, we must choose diet D'_2 and growth period of x days which maximises the revised expected gross margin

$$\max_{D_2} \max_{x} \; c(x) \left\{ \sum_{j=1}^{J} p'_j GR_j(D'_1, D_2; x) - FC_1(D'_1; x) - FC_{2i}(D_2; x) - WC \right\}$$

Note that evaluation of the first stage objective function, for a given F and x, involves $I + 1$ linear programs. Maximisation with respect to x, for fixed F, is carried out pointwise, while maximisation with respect to (D_1, D_2) can be carried out using either a genetic algorithm or the Tailored method. Evaluation of the second stage objective function, for a given $F = (D'_1, D_2)$ and period x, involves just two linear programs; maximisation with respect to x, for fixed $F = (D'_1, D_2)$, is again pointwise and the genetic algorithm or Tailored method used again to find the optimal F.

In the case of $K > 2$ diets, the optimisation proceeds in K stages, with the mth ($m = 1, \ldots, K - 1$) stage involving solution of (with notation extending that above in a ready way)

$$\max_{F} \max_{x} \; c(x) \left\{ \sum_{j=1}^{J} p'_j GR_j(F; x) - \sum_{k=1}^{m-1} FC_k(D'_k; x) - \sum_{s=m}^{K} \sum_{i=1}^{I} p_i FC_{si}(D_s; x) - WC \right\}$$

Table 5 Price schedule 2 (PS_2): a generated price schedule giving prices in cents/kg for pigs at slaughter

	Carcass weight (kg)										
Backfat (mm)	35.0 and under	35.1 – 40.0	40.1 – 45.0	45.1 – 50.0	50.1 – 55.0	55.1 – 60.0	60.1 – 65.0	65.1 – 70.0	70.1 – 75.0	75.1 – 80.0	80.1 and over
<6	300	300	300	300	300	300	300	300	300	300	300
6–9	335	345	350	365	370	375	380	390	395	405	375
10–12	335	345	350	360	365	370	375	385	390	400	370
13–15	305	330	335	335	335	335	330	330	330	330	330
16–18	270	270	270	270	270	270	260	260	260	260	260
>18	240	240	240	240	240	240	230	230	230	230	230

where $F = (D'_1, \ldots, D'_{m-1}, D_m, \ldots, D_K)$, and the Kth stage involving solution of

$$\max_{F=(D'_1,\ldots,D'_{K-1},D_K)} \max_x c(x) \left\{ \sum_{j=1}^{J} p'_j GR_j(F;x) - \sum_{k=1}^{K-1} FC_k(D'_k;x) - FC_{Ki}(D_K;x) - WC \right\}$$

4.1 Numerical results

Uncertainty about the price schedule is easily handled, in that the optimisation uses the p'_1 and p'_2 weighted convex combination of schedules P_1 and P_2 respectively. For completeness, in our numerical runs we used two such schedules, those shown in Tables 2 and 5, with associated weights $p'_1 = 0.8$ and $p'_2 = 0.2$. All optimisations were carried out using a genetic algorithm.

Uncertainty in the ingredient schedule is of much greater interest and for these we use the ingredient schedules in Table 1 with weights of $p_1 = 0.8$ and $p_2 = 0.2$. Optimal feeding schedules (with two diets fed) are shown in Table 6 when there is no uncertainty in the diet to be fed for the second period and in Table 7 when the digestible energy cheap ingredient schedule IS_1 is far more likely ($p_1 = 0.8$) than the protein cheap ingredient schedule ($p_2 = 0.2$) in the second feeding period. Pig genotype parameters of $Pd_{\max} = 160$ and $\min LP = 0.8$ are used.

Some comments about the results are now provided. In Table 6, cheap energy (carbohydrate) via use of IS_1 in the first period allows the proportion of the NRC standard used in the second period to drop to 0.83 (for either IS_1 or IS_2), whereas it remains higher at 0.87 and 0.88 (for IS_1 and IS_2 respectively) if energy is expensive (use of IS_2) in the first period. No matter which ingredient schedule is used in the first period, the move to cheaper protein, from IS_1 to IS_2, in the second period causes r_2 to increase, as expected. In Table 7, first period diets, for a given ingredient schedule in the first period, do not vary with second period ingredient schedule, again as expected. The first comment made concerning Table 6, regarding p, still largely stands, although it is moderated due to the uncertainty in the second ingredient schedule. The most notable change from the deterministic to stochastic result tables is the larger r_2 value when IS_2 is used in the first feeding period and IS_1 in the second. Here the possibility of cheap protein did not eventuate, so more must be taken in the second feeding period.

Table 6 Deterministic results: optimal feeding schedules when there is no uncertainty about the ingredient schedule in the second feeding period

Optimal feeding schedules		2nd period			
		IS_1		IS_2	
1st period	Parameter	D_1	D_2	D_1	D_2
IS_1	p	0.95	0.83	0.94	0.83
	r	0.64	0.50	0.65	0.52
	d	12.10	14.40	12.10	13.82
IS_2	p	1.00	0.87	1.00	0.88
	r	0.98	0.91	0.99	1.01
	d	15.02	14.80	15.01	15.02

Entries are averages over ten runs

Table 7 Stochastic results: optimal feeding schedules when there is uncertainty about the ingredient schedule in the second feeding period

Optimal feeding schedules		2nd period			
		IS_1		IS_2	
1st period	Parameter	D'_1	D'_2	D'_1	D'_2
IS_1	p	0.97	0.81	0.97	0.81
	r	0.69	0.52	0.69	0.55
	d	12.48	13.99	12.48	12.31
IS_2	p	0.98	0.88	0.98	0.88
	r	0.98	0.99	0.98	1.00
	d	15.08	15.00	15.08	15.01

Entries are averages over ten runs

5 Summary and discussion

Two of the many challenges which exist in the finding of optimal animal diets have been addressed in this paper. First, we have examined the nature of the objective function, found it to provide a single but very rough peak, and so tailored a heuristic algorithm to its shape. This algorithm climbs quickly and appears to find better optima than previous methods. Second, we have considered variation in the ingredient schedule and price schedule, and shown how to find optimal feeding schedules under such conditions.

Although two or three feeding periods is the norm in production units today, in future, with increased use of computerised feeding of large production units, it will become feasible to change diets more regularly. The methods of this paper can be applied, but the dimension of the problem will increase. The performance of the Tailored method in such problems remains to be investigated.

We caution that the optimal feeding schedule for a single pig is unrealistic, since in practice many pigs, exhibiting minor variations in genotype and feed intake, are grown on a single feeding schedule. The optimum schedule in such a situation is different from that found for a single pig. Such variation can be incorporated into an objective function, but was not in this paper, in order to focus on the two developments addressed.

We conclude by acknowledging that there will always remain scope for improved methodology in this rich application area for optimisation.

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